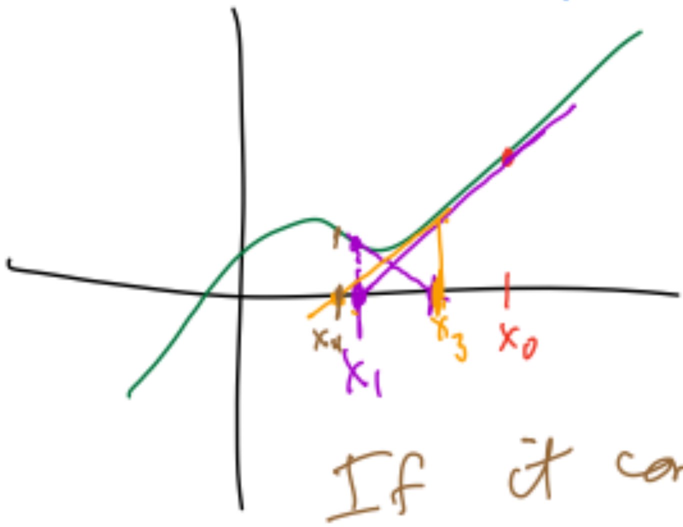
Newton-Raphson Method of solving $f(x)=0$.

- Start with initial guess x_0
- $x_{j+1} = x_j - \dfrac{f(x_j)}{f'(x_j)}$ for $j \geq 0$.



If it converges (ie if initial guess is close enough),

then it has order 2 convergence (very fast!)

ie if x_* is the root we are trying to find, so $f(x_*)=0$, and

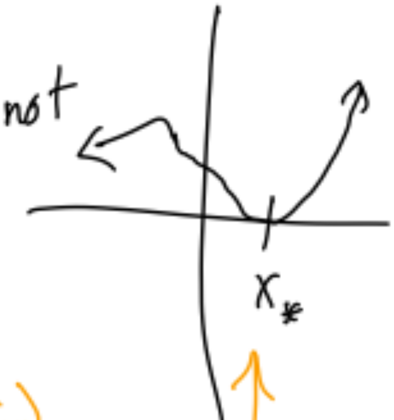
$$e_j = x_* - x_j = j^{\underline{th}} \text{ error}.$$

then $\dfrac{|e_{j+1}|}{|e_j|^2}$ is bounded as j increases.

☆ This is true if f is differentiable, in fact C^2 (2^{nd} exists & is continuous), and if $\underbrace{f'(x_*) \neq 0}$

↑ ie the zero is of order 1

, not



here

$f(x) \simeq f'(x_*)(x - x_*)$

(1st degree term in Taylor)

$f(x) \simeq \frac{f''(x_*)}{2!}(x - x_*)^2$

higher degree zero of order k: $f(x) \simeq \frac{f^{(k)}(x_*)}{k!}(x - x_*)^k$

Fact: If x_* is a zero of higher order (degree k), then the order of convergence of the N-R method is just 1.

But: if we know the order of the zero.

We can use the Modified NR method:

$$\boxed{\begin{array}{l} x_0 \text{ initial guess} \\ x_{j+1} = x_j - k\dfrac{f(x_j)}{f'(x_j)} \\ \qquad\qquad j \geq 0 \end{array}}$$

Modified NR method

zero of order k.

converges with order 2.

Another fun fact: the secant method has order $\frac{1+\sqrt{5}}{2}$ (the golden ratio).